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INFLUENCE OF THE AQUEOUS-PHASE PRESSURE ON THE FREEZING AND THAWING OF PORE MOISTURE IN HIGHLY DISPERSE MEDIA

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The freezing and thawing of sandy (frozen) rock is considered, taking account of the temperature and pressure correlation at the phase-transition front.

To ensure reliable construction and operation of boreholes in extreme northerly regions, it is necessary to know the laws of rock freezing. In some models of highly disperse media, for example sandy loam, sand, sandstone, thawing and freezing are described by the Stefan problem [1, 2], in which the existence of a smooth frontal surface between two phases of pore water — liquid and solid — is assumed. The temperature at the front is usually assumed to be constant throughout the whole period of development of the process. However, in situations that are of practical interest, it is common to determine the pressure in the liquid phase, which is transmitted there from outside or created there because the front displaces the water excess as a consequence of the density difference between the two phases. In a limited volume of water phase, the pressure arising on freezing may contort the borehole

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column [3, 4]. If the volume of water phase is not closed, the pressure is determined by the filtrational characteristics of the porous medium.

Investigation of the Stefan problem for water evaporation, taking account of the dependence of the temperature on the pressure at the phase-transition front, reveals [5, 6] a series of interesting effects. It is natural to use an analogous approach in investigating phase transition at the liquid–solid boundary.

Of the many problems of this type arising in practice, the three considered below share a common method of solution and character of occurrence of the process (plane-parallel symmetry in heat and liquid transmission and also the simplest boundary conditions, permitting a self-similar formulation).

It is assumed here that, with increase in pressure on the ice, its melting point will decrease in accordance with the Clausius–Clapeyron equation. With change in pressure to 1000 bar, this dependence may be regarded as linear without great error

$$p_m - p_0 = p(T_m) - p(T_0) = \delta(T_0 - T_m), \quad \delta = 134 \text{ bar/K}, \tag{1}$$

where p_0 and T_0 are the pressure and temperature in standard conditions.

In addition, it is taken into account that in a system consisting of water and ice the additional formation of ice in the volume V_I leads to increase in water volume by an amount ΔV_I; $\Delta = (\rho_W - \rho_I)/\rho_W$. In the inverse process – melting of ice in the volume V_I – the water in the system decreases by the same amount ΔV_I; ρ_W, ρ_I are the densities of water and ice, respectively, and are assumed to be constant below.

1. The first problem investigated concerns the increase in pore pressure under the phase-transition front with omnidirectional freezing of a completely water-saturated porous medium occupying the half-space $x > 0$. The permeability k and porosity m of the medium are regarded as constant, as are the water viscosity μ and the compressibility of the pore space β in the water-saturated part of the medium. The frozen zone which forms is regarded as undeformable. It is also assumed that the thermal conductivity λ_i and thermal diffusivity α_i ($i = 0, 1$) do not change within the limits of the thawed and frozen zones. The temperature T_s at the surface of the half-space $x = 0$ remains constant throughout the whole process, and the temperature T_m at the phase-transition front with current coordinate $x_m(t)$ changes only as a function of the pressure p_m directly beneath the front.

The temperature distribution in the zones of the growing ($0 \leqq x \leqq x_m$) and initial ($x_m \leqq x \leqq \infty$) phases may be described by linear heat-conduction equations

$$\alpha_1 \frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_1}{\partial t}, \quad \alpha_0 \frac{\partial^2 T_0}{\partial x^2} = \frac{\partial T_0}{\partial t}$$

with initial and boundary conditions

$$x = 0 \quad T_1 = T_s; \quad x = \infty \quad T_0 = T_i; \quad t = 0 \quad T_0 = T_i; \quad x_m(0) = 0;$$

$$x = x_m(t) \quad T_1 = T_0 = T_m; \quad -\lambda_1 \frac{\partial T_1}{\partial x} = m\rho_W L \frac{\partial x_m}{\partial t} - \lambda_0 \frac{\partial T_0}{\partial x}.$$

In the water-saturated zone, the change in filtration potential $\varphi = p + \rho_W g x$ is determined by the piezoconduction equation

$$\varkappa \frac{\partial^2 \varphi}{\partial \varkappa^2} = \frac{\partial \varphi}{\partial t}, \quad \varkappa = \frac{k}{\mu\beta}$$

with the initial condition $\varphi(x, 0) = \varphi_i = p_0 + \rho_W g x$.

The additional condition for solving the heat-conduction equations specifies the filtration rate at the freezing point

$$x = x_m \quad -\frac{k}{\mu} \frac{\partial \varphi}{\partial x} = \Delta m \frac{dx_m}{dt}.$$

This problem reduces to self-similar form by means of the Boltzmann substitution $\xi = x/2\sqrt{\alpha_1 t}$. The temperature distribution in the zones of growing ($i = 1$) and initial ($i = 0$)

phases is expressed in terms of the error function erf ξ and the complementary function cerf ξ in the following form:

$$\frac{T_1 - T_m}{T_s - T_m} = 1 - \frac{\operatorname{erf}\xi}{\operatorname{erf}\xi_m}, \quad \frac{T_m - T_i}{T_i - T_m} = -1 + \frac{\operatorname{cerf}\xi\sqrt{\beta}}{\operatorname{cerf}\xi_m\sqrt{\beta}},$$

$$\beta = \frac{\alpha_1}{\alpha_0},$$

where $\xi_m = x_m/2\sqrt{\alpha_1 t}$; $x_m = \sqrt{2\lambda\alpha_1 t}$, and the constant λ, which characterizes the law of front motion, is determined from the heat-balance equation at the front. Then

$$\frac{K_1}{J_1(\lambda)} - \frac{K_0}{J_0(\lambda\beta)} = \lambda, \tag{2}$$

$$K_1 = \frac{\lambda_1 (T_m - T_s)}{\alpha_1 \beta_W L_m}, \quad K_0 = \frac{\lambda_0 (T_m - T_i)}{\alpha_1 \rho_W L_m},$$

$$J_1(\lambda) = \sqrt{\frac{\pi}{2\lambda}}\, e^{\lambda/2} \operatorname{erf}\sqrt{\frac{1}{2}}\lambda, \quad J_0(\lambda) = \sqrt{\frac{\pi}{2\lambda}}\, e^{\lambda/2} \operatorname{cerf}\sqrt{\frac{1}{2}}\lambda.$$

The functions introduced here are useful in solving self-similar problems of Stefan type as a result of the simple approximations found in this case

$$\frac{1}{J_1(\lambda)} \simeq \sqrt{1 + \frac{1}{3}\lambda e^{\lambda/2}};$$

$$\frac{1}{J_0(\lambda)} = \frac{2}{\pi}\lambda + \sqrt{\frac{2}{\pi}\lambda + \left(\frac{\pi}{2} - 1\right)^2 \left(\frac{2}{\pi}\lambda\right)^2}.$$

The Boltzmann substitution for the piezoconduction equation leads to the following expression for the pressure increase at the phase-transition front:

$$\frac{p_m - p_0}{p_0} = M\lambda\alpha J_0(\lambda\alpha), \tag{3}$$

$$\alpha = \frac{\alpha_1}{\varkappa}, \quad M = \frac{m}{\beta\rho_0}\Delta.$$

Thus, three equations – Eqs. (1)-(3) – are found for determining three unknowns: λ, p_m, and T_m. They are found most simply by the method of successive approximation. Thus, specifying an appropriate value of T_m, the parameters K_1 and K_0 are determined; then the root of Eq. (2) is found and hence the increase in pressure at the front according to Eq. (3). Then the next approximation for T_m is found from Eq. (1) and the successive-approximation procedure is repeated until the value of T_m obtained coincides with that in the previous iteration, within the limits of the specified error.

The pressure increment at the phase-transition front is estimated as a function of the bed permeability with the following parameter values (Fig. 1): thermal conductivity in the thawed and frozen zones 1.75 and 2 W/m·K, respectively; thermal diffusivity in the same zones $0.9 \cdot 10^{-6}$ and $0.6 \cdot 10^{-6}$ m²/sec, respectively; latent heat of phase transition $331.6 \cdot 10^3$ J/kg; bed porosity m = 0.2; effective compressibility of the pore space $\beta = 5 \cdot 10^{-5}$ bar^{-1}; density of ice and water $\rho_I = 917$ and $\rho_W = 997.97$ kg/m³; water viscosity $\mu = 10^{-3}$ Pa·sec; initial bed temperature 2°C; temperature at surface $T_s = -5$°C.

A sandy bed corresponds to $k \geqq 10^{-3}$ μm². The pressure increment at the front is slight in this case and has virtually no influence on the phase-transition temperature. Therefore, there is no need to repeat the iteration.

The need for the iterative process only arises when the permeability of the medium is slight. Physically, this corresponds to the case when the sink region is partitioned off from the freezing sandy bed by a water-tight barrier of permeability $k = 10^{-3}$ μm². In view of the small filtrational resistance in the sandy bed itself, the same formulas may be used to determine $\Delta p = p_m = p_0$.

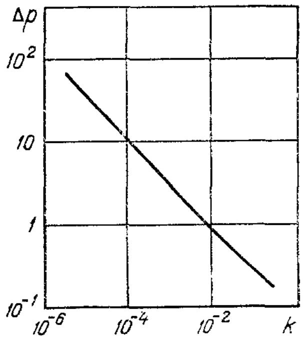

Fig. 1. Pressure increment Δp at freezing front as a function of permeability of disperse medium k.

TABLE 1. Values of $a = \sqrt{\lambda/2}$ from Eq. (2)

p_s, bar	T_m, °C	K_1	$-K_0$	a	a/a_0	ΔT, °C
0	0	0,0477	—0,0273	0,149	1	0
134,3	—1	0,0954	0,0273	0,225	1,51	3,2
268,4	—2	0,1431	0,0819	0,285	1,91	4,6
403,2	—3	0,1908	0,1365	0,341	2,29	6,3

Freezing of beds with low permeability or at very small depths may occur with deformation of the roof, which breaks apart the soil grains and fills the intervening space with ice. Thus, within the limits of massive cryogenic texture, swelling of the soil occurs, with the formation of excess ice beyond the limits of the pore volume in the thawed state. In the case of a barrier positioned close to the earth's surface, deformation of the roof may appear in disruptive dislocations at the weakest points. This may possibly be the explanation for the natural process of the emergence of ice columns from the earth, which has sometimes been observed. On reaching a height of 3-5 cm, such columns usually tip over, revealing a channel filled with wet soil. The narrow channel rapidly freezes, forming another column, and increase in pressure beneath it culminates in the expulsion also of this column.

Conversely, in the presence of sink regions, which are thawed regions under the banks of rivers and the beds of lakes, when the permeability of the sand is not too small, freezing to great depths will occur without deformation of the roof and the formation of excess ice. This conclusion is confirmed by investigating the core removed from seven special boreholes in the Medvezh'ii and Urengoiskii fields.

2. Now consider the melting of pore ice in a horizontal sandy bed with specified values of the temperature T_s and pressure p_s at the end surface $x = 0$. This situation arises, for example, when water leaks through the nonhermetic columnar space of a borehold from a deep-lying stratum in the frozen-rock interval. The other assumptions made are the same as in the preceding problem. However, in contrast to that problem, the growing phase here is water and the initial phase is ice in the pore space.

The temperature distribution in each of the zones is represented by the same functions, and the filtration potential satisfies the same piezoconduction equation. Since the bed is horizontal, this potential may be identified with the pressure, the distribution of which in the zone $0 \leqq x \leqq x_m(t)$ is determined by the following function, in view of the assumptions adopted:

$$p = p_s - (p_s - p_m)\,\mathrm{erf}\,\frac{x}{2\sqrt{\varkappa t}} \Big/ \mathrm{erf}\,\frac{x_m}{2\sqrt{\varkappa t}}. \tag{4}$$

It follows from the requirement that this function satisfy, as before, the condition of matching of the two fields (temperature and pressure) at the moving front $x_m = \sqrt{2\lambda\alpha_1 t}$ that

$$\frac{p_s - p_m}{p_0} = M\lambda\alpha J_1(\lambda\alpha). \tag{5}$$

Here λ satisfies the same equation, Eq. (2), in which

$$K_1 = \frac{\lambda(T_s - T_m)}{\alpha_1\rho_I L_m};\quad K_0 = \frac{\lambda_0(T_m - T_i)}{\alpha_1\rho_I L_m}.$$

Finally, three equations – Eqs. (1), (2), and (5) – are obtained for determining the three unknowns λ, p_m, and T_m. This system is solved by the method of successive approximation.

The results of solution for a bed with permeability $k = 200\ \mu m^2$ and water temperature $T_s = 1°C$ at the inlet, with various pressures p_s, are shown in Table 1. The initial bed temperature is taken to be $T_i = -0.5°C$, while the other parameters are the same as before.

For comparison with the solution of the Neumann problem, the first row in Table 1 gives the value of $\alpha = \sqrt{\lambda/2}$ when there is no excess pressure on the bed. The quantity α/α_0 is the increase in melting rate of the pore ice under pressure in comparison with the melting rate under ordinary conditions. In the last column, the increase in temperature at the inlet surface of the bed required in the melting of ice without the application of pressure to the liquid phase in order to obtain the same effect is shown.

Investigations show that melting of pore ice under pressure occurs fairly rapidly. Note that, in the given problem, the melting point may be less than the initial value, i.e., $T_m < T_i$, and hence the heat flux from both zones is directed toward the front. This distinguishes the present problem from the classical Neumann problem, in which the heat flux is directed only in one direction. However, whereas in the Neumann problem the solution of Eq. (2) is possible with any values of K_1 and K_0, when $T_m < T_i$, Eq. (2) only has a solution when $K_0\beta < 1$ but for any K_1, including $K_1 = 0$. This is important for the solution of the next problem.

3. Now consider the case when the borehole is surrounded by a deep cavern, in the cross section of which there is a sandy bed; the pores of this bed are initially filled with ice. The cavern is constantly filled with water. In winter, half the cavern freezes and is sealed off by an ice plug of variable thickness $x_0(t)$ (Fig. 2). Suppose that the mean ambient temperature in winter is T_a and the initial temperature of the sandy bed is T_i: $T_a < T_i < 0°C$. On account of the growth of the ice layer at the earth's surface the pressure in the cavern rises, reaching a value p_s which exceeds the equilibrium value for temperature T_i. For this reason, melting of the pore ice at temperature T_m occurs in the sandy bed at a front with a velocity determined by the intensity of heat input from the ice.

In this problem, the cavern is assumed to be plane, the pliability of its walls is completely neglected, and the assumptions regarding the horizontal sandy bed are as before. Under these assumptions, the steady pressure in the cavern p_s is found. It is larger than p_m at the front by an amount due to the filtrational resistance in the thawed part of the bed $0 \leqq x \leqq x_m(t)$.

A constant pressure in the cavern is obviously established in the case where the volume of the water excess generated at the lower end of the ice plug sealing the neck of the cavern is equal to the volume filtering in the bed, i.e., the following condition must hold:

$$A_0\Delta \frac{dx_0}{dt} = -A\frac{k}{\mu}\left.\frac{\partial p}{\partial x}\right|_{x=0}, \tag{6}$$

where A_0 and A are the cross-sectional areas of the cavern neck and the inlet surface of the sandy bed, while $p(x, t)$ is the pressure distribution in the thawed part of the bed.

Assuming a steady pressure distribution over the height of the ice plug and also a steady temperature there, the Leibenzon method gives

$$x_0 = \sqrt{2\frac{\lambda_I(T_m - T_a)}{\rho_W L}t}. \tag{7}$$

As may readily be established, this problem is self-similar. Then the pressure distribution in the thawed part of the bed is specified by Eq. (4), the front motion is determined by the law $x_m = \sqrt{2\lambda\alpha_0 t}$, and the constant λ is the solution of the transcendental equation

$$K_0 = \lambda J_0(\lambda), \quad K_0 = \frac{\lambda_0(T_i - T_m)}{\alpha_0\rho_I L_m}. \tag{8}$$

After minor manipulations, the following equation is obtained from Eqs. (6) and (7):

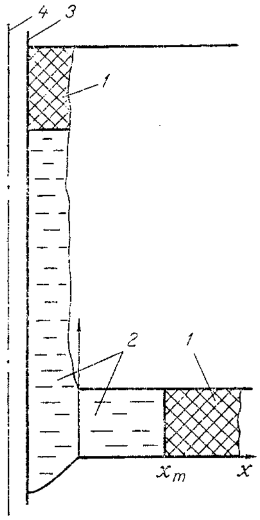

Fig. 2

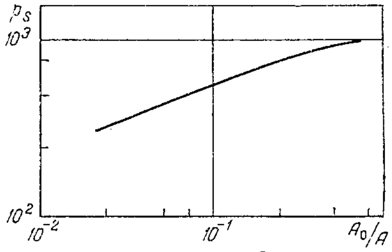

Fig. 3

Fig. 2. Freezing of water in a cavern surrounding a borehole with simultaneous thawing of the pore ice in a sandy bed: 1) ice; 2) water; 3) borehole column; 4) borehole axis.

Fig. 3. Pressure in cavern as a function of the area ratio of the neck and the inlet surface of a sandy bed with a permeability k = 200 μm^2.

$$\left(\frac{A_0}{mA}\right)^2 \frac{\lambda_I}{\varkappa} \frac{T_m - T_a}{\rho_W L} = \lambda\alpha' e^{\lambda\alpha'}, \quad \alpha' = \frac{\alpha_0}{\varkappa}, \tag{9}$$

which, together with Eq. (8), allows the unknowns λ and T_m to be determined. The solution may be somewhat simplified if the unknown temperature at the front and in the cavern T_m is eliminated from Eqs. (8) and (9). Then the following equation is obtained for determining λ:

$$K_a = \lambda\left[J_0(\lambda) + Ne^{\lambda\alpha'}\right],$$
$$K_a = \frac{\lambda_0 (T_i - T_a)}{\alpha_0 \rho_I L}, \quad N = \left(\frac{mA}{A_0}\right)^2 \frac{\lambda_0}{\lambda_I} \frac{\rho_л}{\rho_W}. \tag{10}$$

The results of the calculations with T_a = −10°C, T_i = −2°C, and the above-noted values of the other thermophysical parameters are shown in Fig. 3 for various A_0/A.

In real situations, the case $A \gg A_0$ is observed and, as is evident from the calculations, the equilibrium temperature, and hence the equilibrium pressure in the cavern, approach values characteristic of a sandy bed with an initial temperature T_i. Therefore, freezing of deep caverns in winter is practically never accompanied by contortion of the column, as noted in borehole-drilling practice in the north of the Tyumen' region and the Krasnoyarsk region.

NOTATION

p, pressure; T, temperature; V, volume; ρ, density; δ, coefficient in Clausius–Clapeyron equation; Δ, relative increment in water volume on phase transition; λ, thermal conductivity; α, thermal diffusivity; m, porosity; k, permeability; ϰ, piezoconduction; μ, viscosity; g, acceleration due to gravity; α, λ, K, x_0, μ, M, dimensionless parameters. Indices: 1, new phase; 0, initial phase; W, water; I, ice; m, phase-transition boundary.

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HEAT AND MASS TRANSFER IN COMMERCIAL DESORBERS WITH A HEATING SURFACE IN THE FORM OF A VERTICAL TUBE BUNDLE AND A THERMOFLUIDIZED HEAT-CARRIER BED

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UDC 66.096.5

A mathematical model is presented of heat and mass transfer in a thermofluidized bed in the thermal regeneration of synthetic zeolites. Theoretical data are compared with experimental results.

This article is a continuation of the work done in [1, 4] on the use of thermofluidization to intensify heat and mass transfer in the regeneration of synthetic zeolites of CaA, MgA, and other aluminates in processing equipment at machine-building plants.

The regime of thermofluidization of a flow of disperse sorbent during its heating begins when evolution of the gaseous sorbate becomes so intense that the velocity of the gas phase exceeds the initial fluidization velocity. The properties of the disperse flow change abruptly at this moment, mixing becomes intense, and there is a significant increase in the effective thermal conductivity of the bed in the thermofluidization zone. It is difficult to calculate the thermofluidization regime due to the nonlinearity of the corresponding model.

Here we use a model of a descending flow moving past a vertical cylindrical heater. We use the approximation that the medium is continuous and has effective properties which model dense and thermofluidized flow. The numerical solutions obtained can then be used to obtain an engineering optimization of commercial units to extract carbon dioxide and other gaseous sorbates from a gas suspension.

We will examine an annular vertical channel (Fig. 1) R_0 with a cylindrical internal heater of radius r_0. The rectangular region BCDE and its longitudinal section are the two-dimensional calculation region, with the coordinate origin at point A. A zeolite flow G crosses section BC with a specified degree of adsorption a_0 and an initial temperature T_0. The boundary CD is thermally insulated and nonpermeable to the gas and solid phase. The boundary BE is heated by the heat flow Q and is also nonpermeable. The porosity ε_0 on the boundary DE is equal to the porosity of the dense flow ($\varepsilon_0 = 0.4$), since there is a gas seal here and gas evolution is directed upward toward section BC. The gas flow increases during ascent and, beginning at a certain section, the bed of sorbent is fluidized.

The sought functions are: the temperature of the flow $T(r, X)$, the porosity of the flow $\varepsilon(r, X)$, and the degree of adsorption $a(r, X)$.

In the approximation of a continuum with uniform adsorption and a negligibly short thermal relaxation time, the flow is described by the system of equations (in the region $r_0 \leqq r \leqq R_0$; $0 \leqq X \leqq X_0$):

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